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On Regular Difilters in Ditopological Texture Spaces

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Abstract

The focus of this paper is to introduce the new spaces namely α- door spaces, α-irreducible,α-Hyperconnectedness which are used to define Regular difilters in di- topological texture spaces. Here we analyze the properties of these notions and obtain some of their characterizations.

Keywords :Ditopology, texture spaces, α- door spaces, hyperconnectedness, α-hyperconnectedness, connectedness, α-irreducible, co-α irreducible
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1 Introduction

L.M.Brown[2] initiated the notion of Textures as a point-set for the study of fuzzy sets in 1998. On the other hand, textures offers a convenient setting for the investigation of complement-free concepts in general. So much of the recent work has been proceeded independently of the fuzzy setting.The concepts of hyperconnectedness, irreducible, door space in topological space were introduced by many mathematicians. This idea is further developed recently by Brown et al to ditopological settings.

In this paper we present some classes of new spaces namely the α- door spaces, α-irreducible, α-hyperconnectedness in dichotomous topologies or ditopology for short.

In Ditopological Texture Spaces: Let S be a set, a texturing T[2] of S is a subset of P(S). If (1) $(T,\subseteq)$ is a complete lattice containing S and φ, and the meet and join operations in $(T,\subseteq)$ are related with the intersection and union operations in $(P(S),\subseteq)$by the equalities

$\bigwedge_{i\in I} A_i = \bigcap_{i\in I} A_i, A_i \in T, i \in I$, for all index sets I, while

$\bigvee_{i\in I} A_i = \bigcup_{i\in I} A_i, A_i \in T, i \in I$, for all index sets I.

(2) T is completely distributive.

(3) T separates the points of S. That is, given $s_1 \neq s_2$ in S we have $A \in T$ with $s_1 \in A, s_2 \notin A$, or $A \in T$ with $s_2 \in A, s_1 \notin A$.

If S is textured by T we call (S,T) a texture space or simply a texture.
For a texture (S; T), most properties are conveniently defined in terms of the p-sets $P_s = \cap\{A \in T / s \in A\}$ and the q-sets, $Q_s = \vee\{A \in T / s \notin A\}$ The following are some basic examples of textures.

Example 1.1 Some examples of texture spaces,

(1) If X is a set and P(X) the powerset of X, then (X; P(X)) is the discrete texture on X. For $x \in X$, $P_x = \{x\}$ and $Q_x = X¥\{x\}$.

(2) Setting I = [0; 1], T= $\{[0;r);[0;r]/r \in I\}$ gives the unit interval texture (I; T). For $r \in I$, $P_r = [0;r]$ and $Q_r = [0;r)$.

(3) $T=\{\varphi, \{a,b\}, \{b\}, \{b,c\}, S\}$ is a simple textureing of $S = \{a,b,c\}$ clearly $P_a = \{a,b\}$, $P_b = \{b\}$ and $P_c = \{b,c\}$.

Since a texturing T need not be closed under the operation of taking the set complement, the notion of topology is replaced by that of dichotomous topology or ditopology, namely a pair (τ, κ) of subsets of T, where the set of open sets τ satisfies
1. $S, \varphi \in \tau$,
2. $G_1; G_2 \in \tau$ then $G_1 \cap G_2 \in \tau$ and
3. $G_i \in \tau$, $i \in I$ then $\vee_i G_i \in \tau$,
and the set of closed sets κ satisfies
1. $S, \varphi \in \kappa$
2. $K_1; K_2 \in \kappa$ then $K_1 \cup K_2 \in \kappa$ and
3. $K_i \in \kappa$, $i \in I$ then $\cap K_i \in \kappa$. Hence a ditopology is essentially a 'topology" for which there is no a priori relation between the open and closed sets.

For $A \in T$ we define the closure [A] or cl(A) and the interior]A[or int(A) under (τ, κ) by the equalities $[A] = \cap\{K \in \kappa / A \subset K\}$ and $]A[= \vee\{G \in \tau / G \subset A\}$:

Definition 1.2 For a ditopological texture space $(S; T; \tau, \kappa)$:

$A \in T$ is called α-open (b-open) if $A \subseteq$ intclintA ($A \subseteq$ clint(A) $\cup$ intcl(A)). $B \in T$ is called α-closed (resp. b-closed) if clintclB $\subseteq$ B (intclB $\cup$ clintB $\subseteq$ B)

We denote by $\alpha O(S; T; \tau, \kappa)$ ($bO(S; T; \tau, \kappa)$), more simply by $\alpha O(S)$ ($bO(S)$) , the set of α-open sets (b-open sets) in S. Likewise, $\alpha C(S; T; \tau, \kappa)$ ($bC(S; T; \tau, \kappa)$), $\alpha C(S)$ ($bC(S)$) will denote the set of α-closed (b-closed sets) sets.

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Definition 1.3 [15] A ditopological space (S, T, τ, κ) is called door if each $A \in T$ either open $A \in \tau$ or $A \in \kappa$.

Definition 1.4 [15] A ditopological space (S, T, τ, κ) is called

1. irreducible if $G_1 \cap G_2 \neq \varphi$ for every $G_1, G_2 \in \tau / \{\varphi\}$
2. co-irreducible if $H_1 \cup H_2 \neq S$ for every $H_1, H_2 \in \kappa / \{S\}$,
3. bi-irreducible if it is irreducible and co-irreducible

Definition 1.5 [1] A difilter on a texture (S,T) is $\mathcal{F} \times \mathcal{G}$, where $\mathcal{F}$ and $\mathcal{G}$ are nonempty and subsets of T satisfies

1. $\varphi \neq \mathcal{F}, F \in \mathcal{F}, F \subseteq F^{0} \in T \Rightarrow F^{0} \in \mathcal{F}$ and $F_1, F_2 \in \mathcal{F} \Rightarrow F_1 \cap F_2 \in \mathcal{F}$
2. $S \neq \mathcal{G}, G \in \mathcal{F}, G \supseteq G^{0} \in T \Rightarrow G^{0} \in \mathcal{G}$ and $G_1, G_2 \in \mathcal{G} \Rightarrow G_1 \cup G_2 \in \mathcal{G}$

Definition 1.6 [1] A difilter $\mathcal{F} \times \mathcal{G}$ is said to be regular if $\mathcal{F} \cap \mathcal{G} = \varphi$ or equivalently, $A \subseteq B$ for every $A \in \mathcal{F}$ and for every $B \in \mathcal{G}$.

2 α- door spaces

Definition 2.1 A topology and co-topology are formed using α-open sets and α-closed sets in Texture space using (τ, κ), such that $\tau \subset \tau_\alpha$ and the α-open sets satisfy
1. $S, \varphi \in \tau\alpha$,
2. If $G_1; G_2 \in \tau\alpha$ then $G_1 \cap G_2 \in \tau\alpha$ and
3. If $G_i \in \tau\alpha$, $i \in I$ then $\bigvee_i G_i \in \tau\alpha$,
and the set of α-closed sets in $\kappa\alpha$ satisfy $\kappa \subset \kappa\alpha$ and
1. $S, \varphi \in \kappa\alpha$
2. If $K_1; K_2 \in \kappa\alpha$ then $K_1 \cup K_2 \in \kappa\alpha$ and
3. If $K_i \in \kappa\alpha$, $i \in I$ then $\cap K_i \in \kappa\alpha$. This new topology for which there is no priori relation between the α-open and α-closed sets.

For $A \in T$ we define the operators $\alpha cl(A)$ and $\alpha int(A)$ under $(\tau\alpha, \kappa\alpha)$ as $\alpha cl(A) = \bigcap \{K \in \kappa\alpha / A \subseteq K\}$ and $\alpha int(A) = \bigvee \{G \in \tau\alpha / G \subseteq A\}$:

Definition 2.2 A ditopological space (S, T, τ, κ) is called α-door if for each $A \in T$ either $A \in \tau\alpha$ or $A \in \kappa\alpha$.

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Remark 2.3 Every door space is α-door space. But the converse need not be true always is shown by the following example.

Example 2.4 Let $S=\{a, b, c\}$, $T=\{\emptyset, \{a\}, \{a, b\}, \{b, c\}, \{a, c\}, X\}$ $\tau = \{\emptyset, X, \{a\}, \{a, b\}\}$ and $\kappa = \{\emptyset, X, \{b, c\}\}$ then $\tau\alpha = \{\emptyset, \{a\}, \{a, b\}, \{a, c\}, X\}$ and $\kappa\alpha=\{\emptyset, \{b, c\}, X\}$ which is not a door space but it is α-door space.

Definition 2.5 A ditopological space (S, T, τ, κ) is called

1. α-irreducible if $G_1 \cap G_2 \neq \emptyset$ for every $G_1, G_2 \in \tau\alpha/\{\emptyset\}$
2. co-αirreducible if $H_1 \cup H_2 \neq S$ for every $H_1, H_2 \in \kappa\alpha/\{S\}$,
3. bi-αirreducible if it is α-irreducible and co-αirreducible

Example 2.6 Let $S=\{a, b, c\}$, $T=\{\emptyset, \{a\}, \{a, b\}, \{b, c\}, \{a, c\}, X\}$ $\tau = \{\emptyset, X, \{a\}, \{a, b\}\}$ and $\kappa = \{\emptyset, X, \{b, c\}\}$ then $\tau\alpha = \{\emptyset, \{a\}, \{a, b\}, \{a, c\}, X\}$ and $\kappa\alpha=\{\emptyset, \{b, c\}, X\}$ This is an example of bi-α-irreducible.

Lemma 2.7 Let $x \in S$ then $\{x\} \in \tau\alpha$ if and only if $\{x\} \in \tau$.
Proof.Straightforward.

Definition 2.8 A ditopological space S is said to α-submaximal if every α dense(i.e) $\alpha cl(A) = S$ subset of S is α open.

Theorem 2.9 Every α-door space S is α-submaximal.

Proof. $A \subset S$ be α dense. If A is not in $\tau\alpha$ then as S is α-door space A is in $\kappa\alpha$. Since A is α closed $\alpha cl(A) = A = S$ which is a contradiction.Thus A is in $\tau\alpha$. (i.e) A is α submaximal.

Corollary 2.10 Every door space S is α-submaximal.

Remark 2.11 Let $(S, T, \sigma, \tau, \kappa)$ be a complemented ditopological space. Then the space is α irreducible if and only if it is co-α irreducible.

Proof. Since the mapping σ is idempotent, $\sigma(\emptyset) = S$ and $\sigma(H_1 \cup H_2) = \sigma(H_1) \cap \sigma(H_2)$ for all $H_1, H_2 \in S$, the proof is clear.

Note 2.12

1. The ditopological space (X,τ) is α irreducible if and only if the corresponding complemented ditopological texture space $(X, P(X), \pi X, \tau, \tau_c)$ is bi α-irreducible.
2. The standard ditopological texture space (I, T, τ_I, κ_I) is not bi α-irreducible. (i.e) It is α-irreducible but not co-α irreducible.

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Definition 2.13 A ditopological texture space (S, T, τ, κ) is called α-hyperconnected if $G \not\subset H$ for every $G \in \alpha O(S)/\{\emptyset\}$ and $H \in \alpha C(S)/\{S\}$, (i.e) $G \in \tau\alpha$ and $H \in \kappa\alpha$.

Example 2.14 Consider the texture (L,T) where L=(0,1] and $T = \{(0, r]¥r \in I\}$. Let $\tau = \{\emptyset, L\}$ and $\kappa = T$. Clearly, this ditopological space is α-hyperconnected.

Remark 2.15 For a complemented ditopological space $(S, T, \sigma, \tau, \kappa)$, having $\sigma(\tau) = \kappa$ and $\sigma(\text{int}(A)) = \text{cl}(\sigma(A))$, we obtain the following implications:

1. $A \in T$ is α dense if and only if $\sigma(A)$ is co-α dense((i.e) α int(A) $= \emptyset$).
2. for every $G \in \tau\alpha/\{\emptyset\}$ is αdense if and only if for every $H \in \kappa\alpha/\{S\}$ is co-αdense.

Proposition 2.16 Let (S, T, τ, κ) be a ditopological space.

1. The space is α-hyperconnected if and only if every $G \in \tau\alpha/\{\emptyset\}$ is αdense.
2. The space is α-hyperconnected if and only if every $H \in \kappa\alpha/\{S\}$ is co-αdense.

Proof. The proof are straightforward.

Theorem 2.17 Let (S, T, τ, κ) is bi α-irreducible and α hyperconnected if and only if $\mathsf{F} \times \mathsf{G}$ is a regular difilter on (S, T), where $\mathsf{F} = \tau\alpha/ \{\emptyset\}$ and $\mathsf{G} = \kappa\alpha / \{S\}$.

Proof. Assume the space to be bi α-irreducible and α hyperconnected and to prove $\mathsf{F} \times \mathsf{G}$ is a regular difilter. Let $F_1, F_2 \in \tau\alpha/\{\emptyset\}$. Since (S, T, τ, κ) αirreducible, $\emptyset \neq F_1 \cap F_2$ and so $F_1 \cap F_2 \in \tau\alpha/\{\emptyset\}$. Now let $F_1 \in \tau\alpha/\{\emptyset\}$ and $F_1 \subseteq F_2$ for some $F_2 \in T$ and $F_2 \in \tau\alpha/\{\emptyset\}$.Thus we proved F to be a filter, similarly we can prove G to be a cofilter, using co-α irreducible. Since it is αhyperconnectedness, from its definition it is clear that no $\alpha O(S)¥\{\emptyset\}$ in $\tau\alpha$ is contained in $\kappa\alpha¥\{S\}$. Thus $\mathsf{F} \times \mathsf{G}$ is a regular difilter.

Let $\mathsf{F} \times \mathsf{G}$ be a regular difilter on (S, T). Then by using the regularity property in the hypothesis, we get (S, T, τ, κ) is αhyperconnected. Let $F_1, F_2 \in \alpha O(S)/\{\emptyset\}$. Since $\alpha O(S)/\{\emptyset\}$ is a filter, $F_1 \cap F_2 \neq \emptyset$, because if $F_1 \cap F_2 = \emptyset$ then $\emptyset \in \mathsf{F}$ which is a contradiction.(i.e.) (S, T, τ, κ) is αirreducible. Similarly we get the (S, T, τ, κ) is co-α irreducible.

Definition 2.18 A regular difilter $\mathsf{F} \times \mathsf{G}$ on (S,T) is maximal if and only if $\mathsf{F} \cup \mathsf{G} = T$ under set inclusion.

Theorem 2.19 Let (S, T, τ, κ) be a ditopological space such that $\mathsf{F} \times \mathsf{G}$ is a maximal regular difilter, where $\mathsf{F} = \tau\alpha/\emptyset$ which is closed under intersection and $\mathsf{G} = \kappa\alpha/S$. Then $(S,T, \tau\alpha, \kappa\alpha)$ is maximal α-hyperconnected and bi-α irreducible ditopological space.

Proof. From regularity and theorem 2.17 $(S,T,\tau\alpha, \kappa\alpha)$ is bi-αirreducible α- hyperconnected ditopological space. Suppose that it is not maximal. Then there exists a bi-αirreducible α hyperconnected space (S, T, τ_1, κ_1) such that $\tau\alpha(S, T, \tau, \kappa) \subseteq \tau_1$ and $\kappa\alpha(S, T, \tau, \kappa) \supseteq \kappa_1$.

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But then $\tau\alpha \subseteq \tau_1\alpha$ and $\kappa\alpha \supseteq \kappa_1\alpha$ which leads to a contradiction to the fact that the difilter given in hypothesis is maximal, since $\tau_1\alpha$ /{ø} is a filter and $\kappa_1\alpha$ /{S} is a cofilter. Therefore (S, T,τα, κα) is bi-αirreducible α- hyperconnected ditopological space is maximal.

Theorem 2.20 Let (S, T, τ, κ) be a ditopological space and $F = \tau/\{ø\}$ and $G = \kappa/\{S\}$. Then (S,T,τ, κ) is bi-αirreducible hyperconnected door space if and only if $F \times G$ is maximal regular difilter on (S,T).

Proof. Suppose that (S,T,τ, κ) is bi-αirreducible hyperconnected door space. We show that F is filter on(S,T). If $G_1, G_2 \in F$ then $ø \neq G_1 \cap G_2 \in F$. Let $G \in F$ and $G \subseteq U$ for some $U \in T$. If U=S then $U \in F$; otherwise $U \in \kappa$, since (S,T,τ, κ) is a door space, then $G \subseteq U$, a contradiction by hyperconnectedness. So F is filter. Likewise, it is obtained that G is a cofilter on (S,T). Clearly, $F \times G$ is a regular difilter, by hyperconnectedness. Furthermore, since (S,T,τ, κ) is a door space, we have $T \subseteq F \cup G$ and so $T = F \cup G$,(i.e) $F \times G$ maximal regular difilter.

conversely, Suppose that $F \times G$ maximal regular difilter on (S,T). Since $F = T/G$then the space is a door ditopological space,by regularity the space is clearly hyperconnected, also it is bi-irreducible since $F \times G$ is a difilter.

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